A Framework for Efficient Dynamic Routing under Stochastically Varying Conditions

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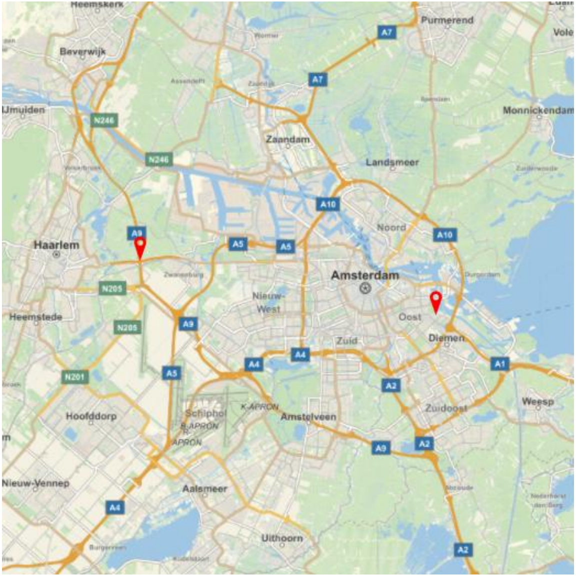
May, 2023

Random Environment



Random Environment

Trip travel times in real traffic networks suffer from:

- Time-dependence: daily patterns, seasonality
- Stochasticity: random effects that lead to delays

Outline

1. Markovian velocity model
2. Find the optimal route
3. Determine the optimal departure time
4. Related and other work

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Consider the graph $G = (N, A)$ with

- N the set of nodes (ramps/intersections)
- A the set of arcs (roads between ramps/intersections)

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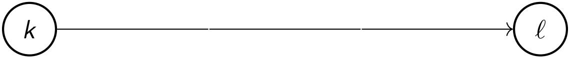

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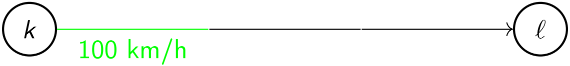

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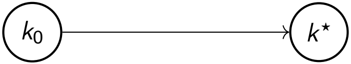
Speeds between ramps vary between finitely many values. We deal with these changing speeds by working with a Markovian background process. This way we can incorporate both time-dependence and random effects.

Markov modulation: random effects



Markov process with incidents:

$$X_{k_0 k^\star}(t) = \begin{cases} 0 & \text{if no incident on road } k_0 k^\star \text{ at time } t \\ 1 & \text{oth.} \end{cases}$$

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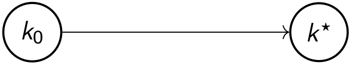

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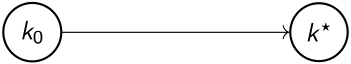

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$v_{k_0 k^\star}(0) = 100$

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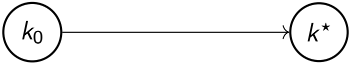

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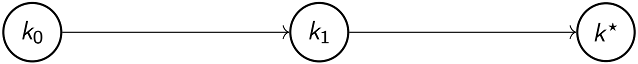

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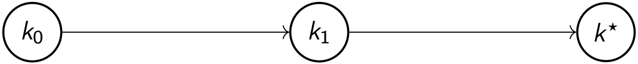

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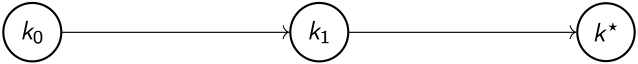

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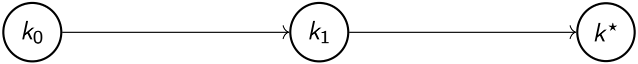

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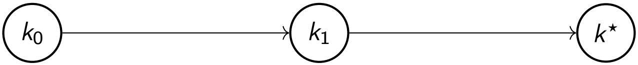

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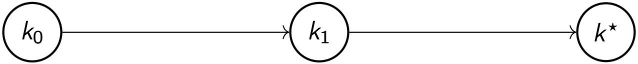

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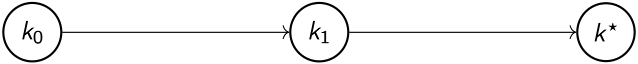

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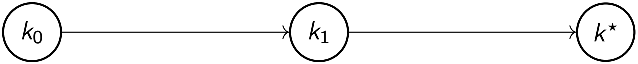

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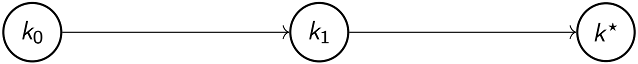

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$$X_i(t) = \begin{cases} 0 & \text{if free-flow speed on road i at time } t \\ 1 & \text{if incident on road } i \text{ at time } t \\ 2 & \text{if recovery state on road } i \text{ at time } t \end{cases}$$

Markov modulation

We can extend our framework by adding more edges.
Notation: $k_1\ell_1, k_2\ell_2, \ldots, k_n\ell_n$.

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Markov modulation: deterministic patterns

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Transition times in our Markov model are exponential:

$$\mathbb{E}[\mathsf{Exp}(\lambda)] = \frac{1}{\lambda} \qquad \mathsf{Var}[\mathsf{Exp}(\lambda)] = \frac{1}{\lambda^2}$$

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$$\mathbb{E}[\sum_{m=1}^{n} \mathrm{Exp}_m(\lambda n)] = \frac{1}{\lambda}$$

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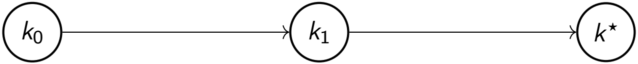
Markov modulation: deterministic patterns



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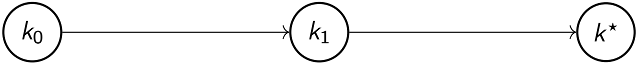

MP with incidents and weather: $B(t) = (X_{k_0 k_1}(t), X_{k_1 k^\star}(t), Y(t))$

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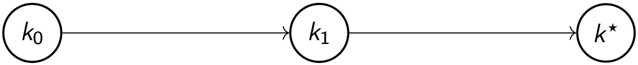

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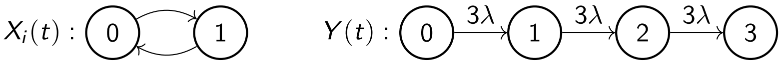

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- Given $Y(t)$, $X_{k_i\ell_i}(t), X_{k_j\ell_j}(t)$ evolve independently;
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Markovian Velocity Model

- Can handle stochasticity
- Can handle time-dependence
- Can handle correlation
- Extreme flexibility (phase-type counterparts)
- Tractability (LST of travel times)

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1. Markovian velocity model
2. **Find the optimal route**: minimize expected travel time
3. Determine the optimal departure time
4. Related and other work

Dynamic routing

Dynamic routing: vehicle is allowed to adapt route

Dynamic routing

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Answer: Yes

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Note: not efficient!

Dynamic Routing

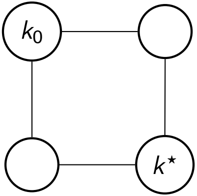


Figure: 2×2-network

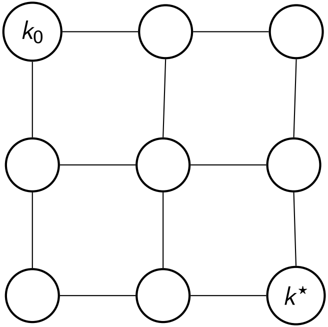
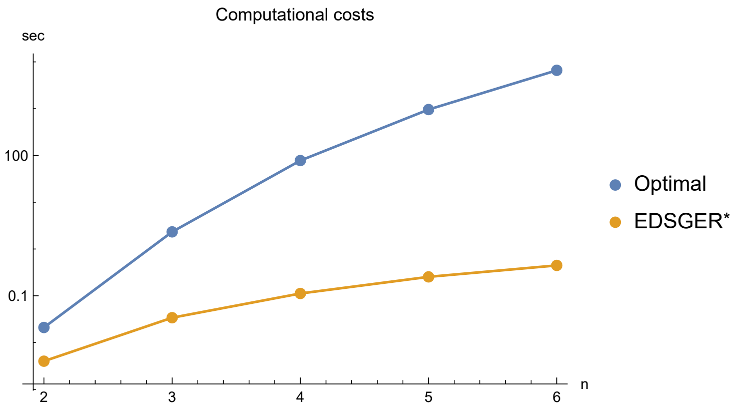


Figure: 3×3-network

Dynamic Routing



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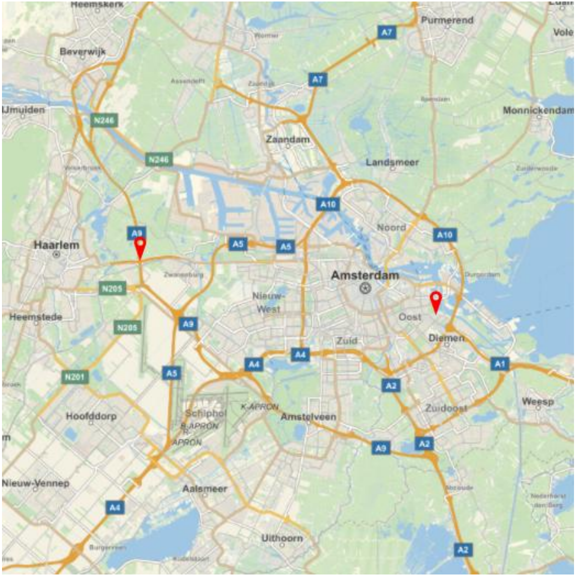
Algorithm for Expected Delay on a Stochastic Graph with Efficient Routing, also known as

EDSGER*.

Figure: Edsger W. Dijkstra

EDSGER*



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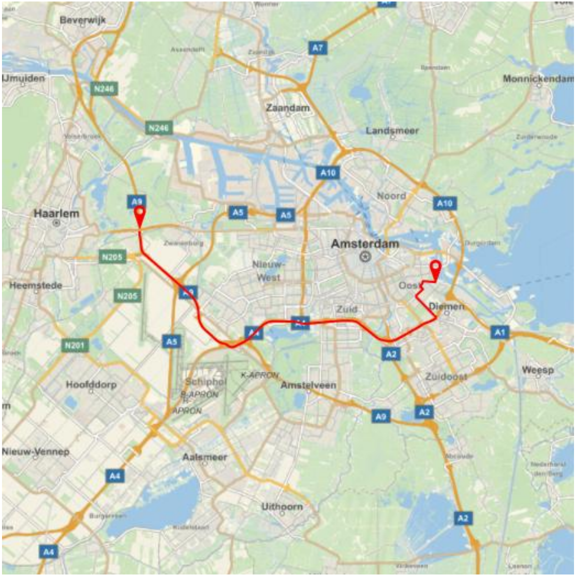
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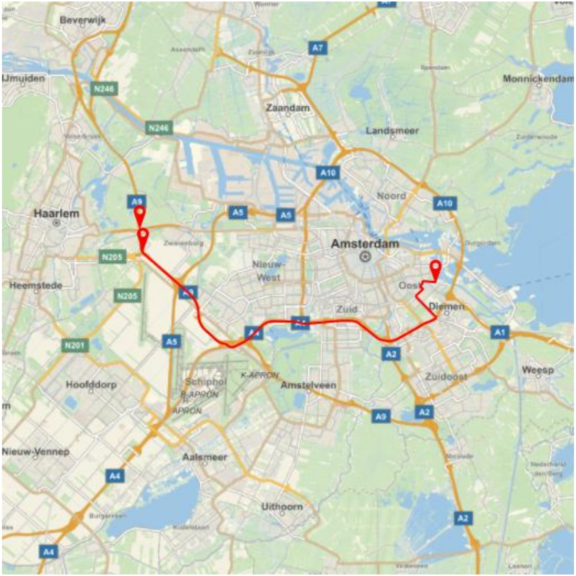
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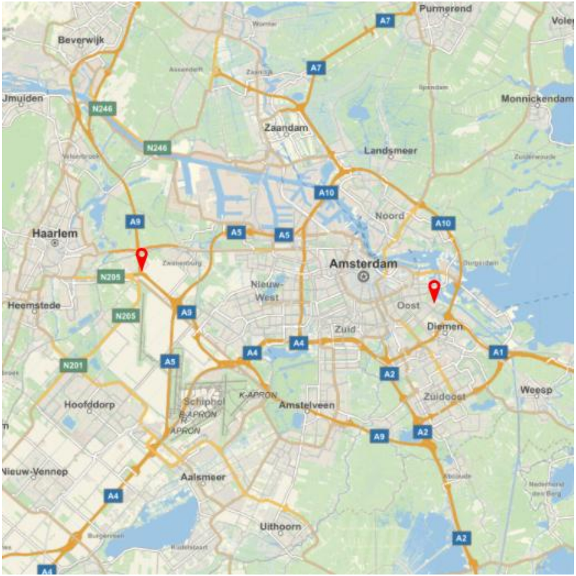

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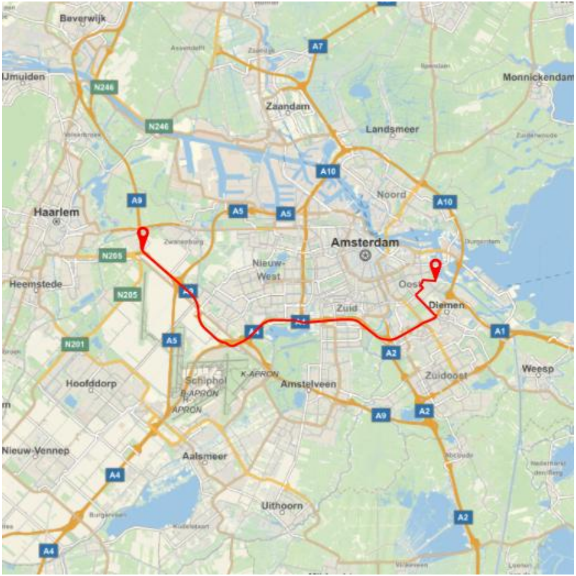

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A2
Nieuw-Vennep
Aalsmeer
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Uithoorn
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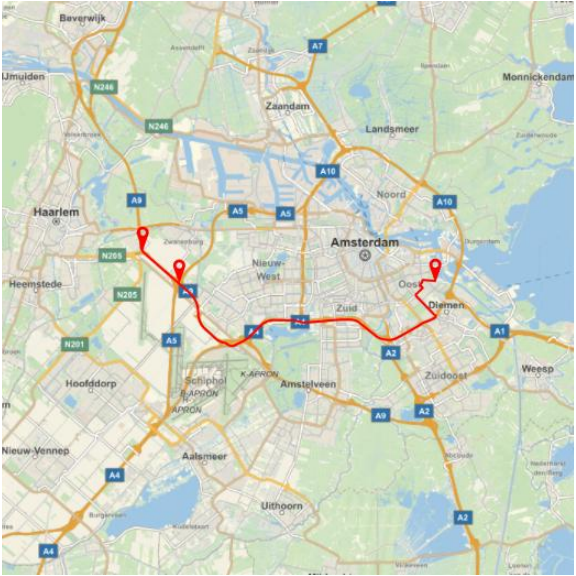
EDSGER*



Beverwijk
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Zaandam
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Noord
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Amsterdam
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Heemstede
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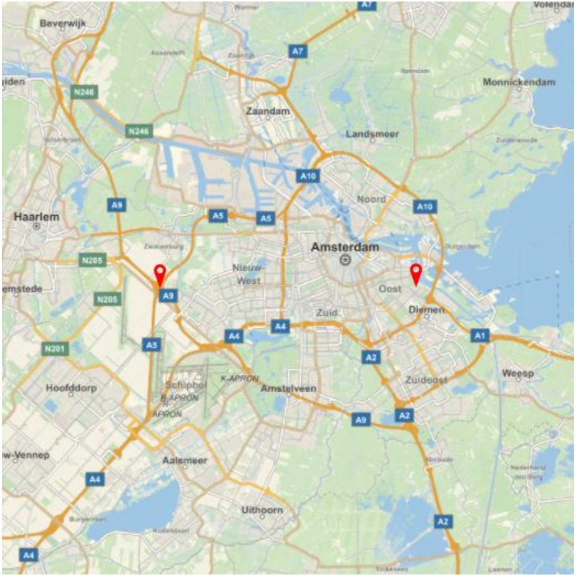

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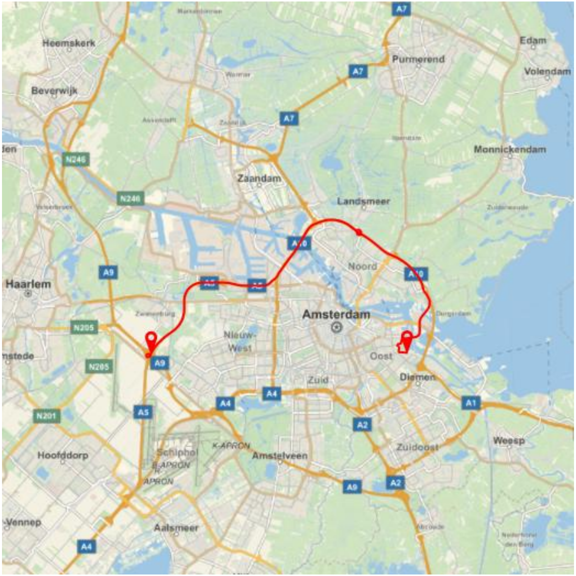
EDSGER*


Heemskerk
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Edam
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EDSGER*

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Shortest path algorithm within EDSGER*: Dijkstra-like

EDSGER⋆

Shortest path algorithm within EDSGER⋆: Dijkstra-like

Assigns labels and iteratively updates the labels of these nodes:

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Shortest path algorithm within EDSGER*: Dijkstra-like

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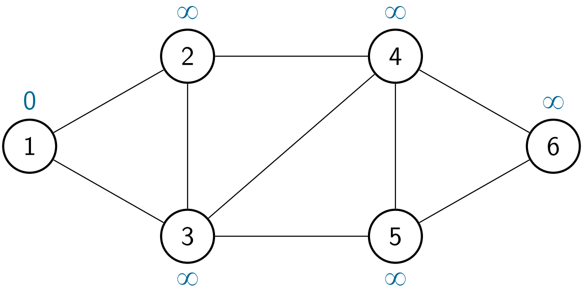

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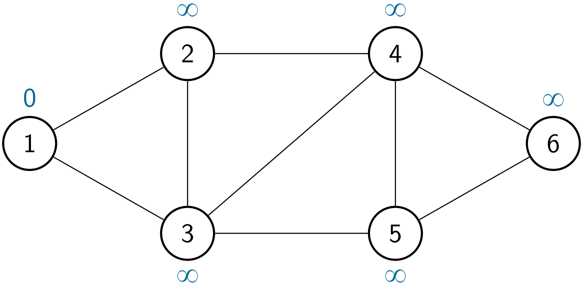

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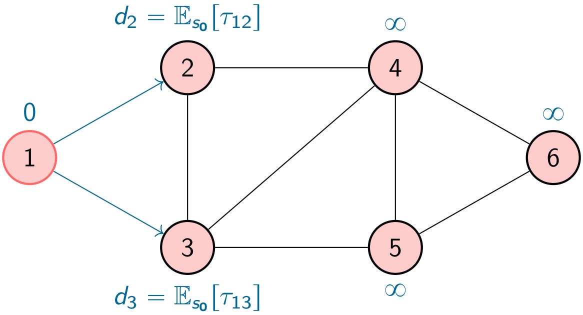
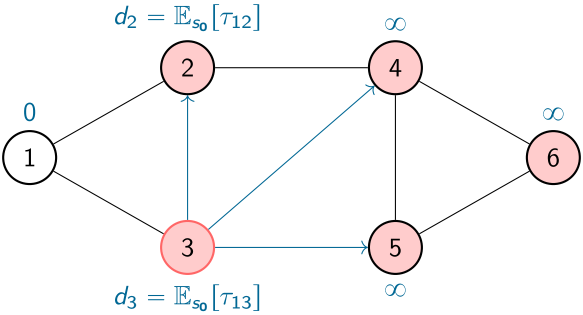


$d_2 = \mathbb{E}_{s_0}[\tau_{12}]$
∞
0
1
2
3
4
5
6
∞
$d_3 = \mathbb{E}_{s_0}[\tau_{13}]$
∞

EDSGER*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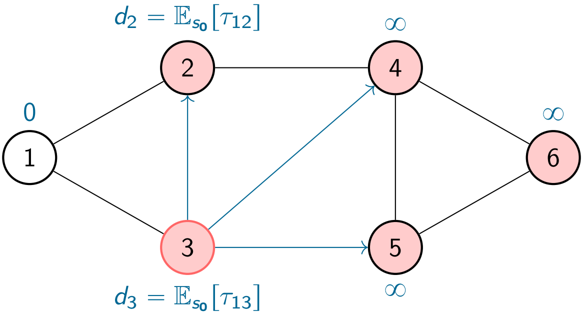

$$d_5 = d_3 + \sum_{s'} p_{s_0 s'} \mathbb{E}_{s'}[\tau_{35}]$$

EDSGER*

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Note:

EDSGER*

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- If $|A| = n$ and every arc has two states, Q has dimension 2^n, which grows exponentially

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Workaround:

EDSGER*

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Workaround:

- local-correlation

Local-correlation

EDSGER*

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Workaround:

- local-correlation: $\dim(Q^{\text{local}}) \ll \dim(Q)$

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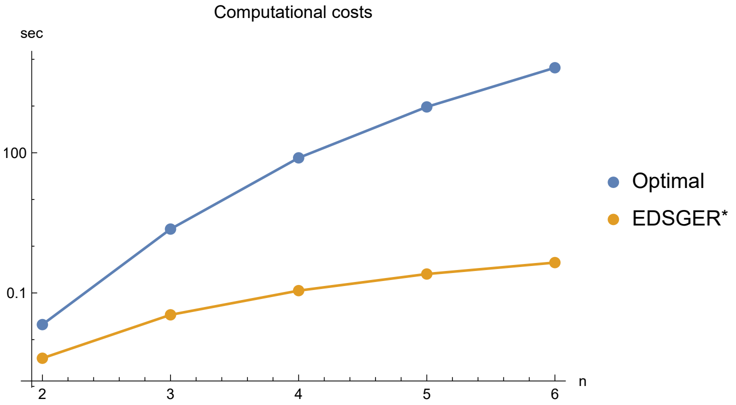
Dynamic Routing



Outline

1. Markovian velocity model
2. **Find the optimal route**
3. Determine the optimal departure time
4. Related and other work

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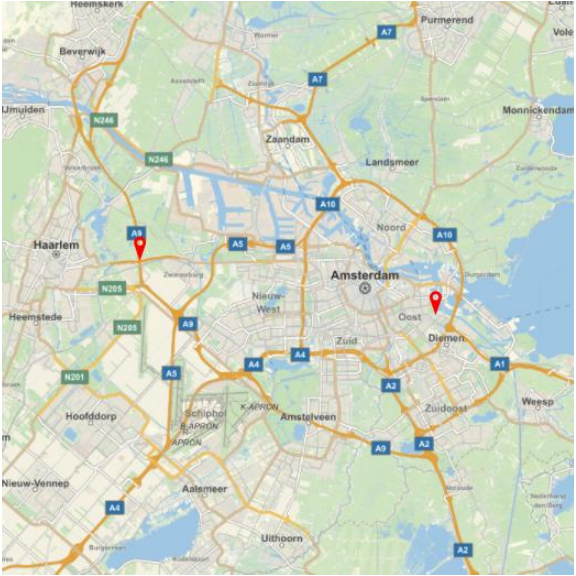

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Optimal departure time:
Latest time of departure for which a chosen on-time arrival probability can be guaranteed.

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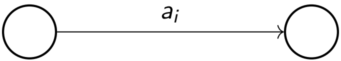
Challenges

Challenges

Trip travel times in real traffic networks suffer from:

- Time-dependence: daily patterns, seasonality
- Stochasticity: random effects that lead to delays

Markovian Velocity Model



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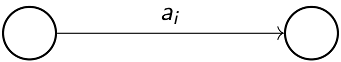

Markov process with incidents:

$$X_{a_i}(t) = \begin{cases} 0 & \text{if no incident on link } a_i \text{ at time } t \\ 1 & \text{oth.} \end{cases}$$

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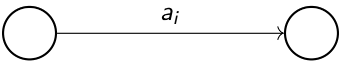

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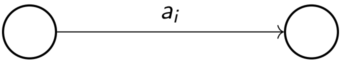

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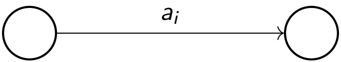
Then, process $X_{a_i}(t)$ is described by its initial state, and Q-matrix:

$$Q_{a_i} = \begin{bmatrix} -\alpha_i & \alpha_i \\ \beta_i & -\beta_i \end{bmatrix} \qquad \alpha_i, \beta_i \in \mathbb{R}_{>0}.$$

Markovian Velocity Model



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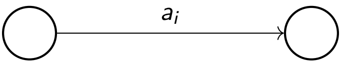

Markov process with incidents:

$$X_{a_i}(t) = \begin{cases} 0 & \text{if no incident on link } a_i \text{ at time } t \\ 1 & \text{oth.} \end{cases}$$

Impact on travel time modeled via vehicle speeds:
Speed at time t equals

- $v_{a_i}(0)$ if $X_{a_i}(t) = 0$
- $v_{a_i}(1)$ if $X_{a_i}(t) = 1$.

Markovian Velocity Model

Markovian Velocity Model

With

$$B(t) := (X_{a_1}(t), \ldots, X_{a_n}(t)),$$

$B(t)$ is a Markovian background process that tracks the occurrence of incidents, and their affect on arc speeds.

Optimal Departure Time Advice

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To determine the optimal departure time, we need:

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Optimal Departure Time Advice

To determine the optimal departure time, we need:

- The desired arrival time M
- The desired on-time arrival probability η
- The current state of the network $B(0)$
- *A path P to travel*

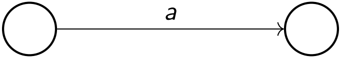
We define the optimal departure time as

$$t_0^* := \sup\{t \geqslant 0 \mid \mathbb{P}\big(t + T_t \leqslant M \mid B(0)\big) \geqslant \eta\},$$

where T_t is the travel time when departing at time t.

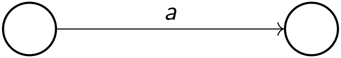
Travel Time Distribution

Travel Time Distribution



1. Immediate departure, one link ($P = \{a\}$, T_0):

Travel Time Distribution



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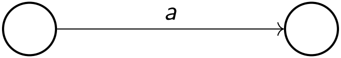

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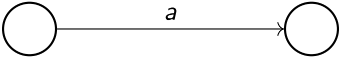

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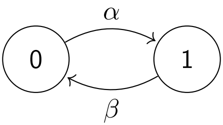
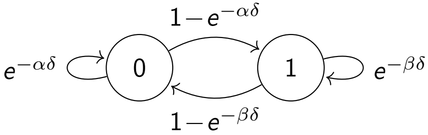
Discretization





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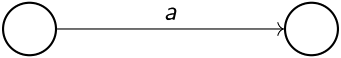

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$$\begin{cases} t_1 & \text{w.p. } p_1 \\ t_2 & \text{w.p. } p_2 \\ \vdots & \end{cases}$$

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Determining the Optimal Departure Time

For each $t \geqslant 0$, we are now able to compute

$$\mathbb{P}\big(t + T_t \leqslant M \,|\, B(0)\big), \quad t \geqslant 0.$$

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$$t' + T'_{t'} \leqslant \tilde{t} + T_{\tilde{t}}.$$

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- Probability monotone decreasing in the departure time
- Bisection algorithm outputs optimal departure time

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So far, we have assumed that the path to the destination is known beforehand.

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For an origin-destination pair:

- Optimal: bisection on output label-correcting algorithm

Optimal Departure Time Advice

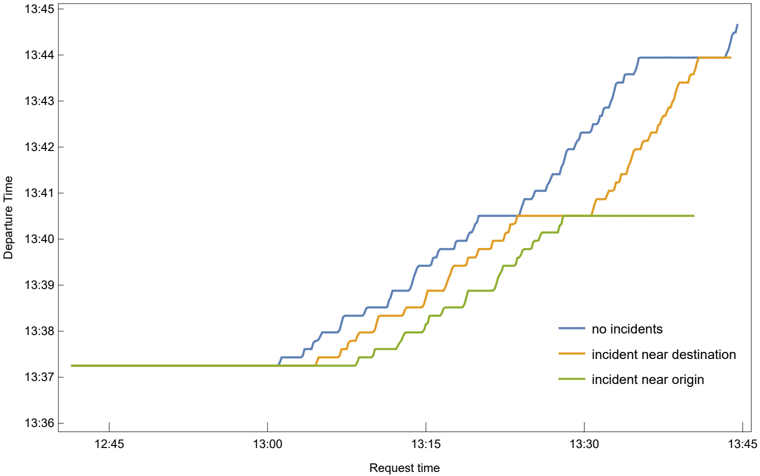
So far, we have assumed that the path to the destination is known beforehand.

For an origin-destination pair:

- Optimal: bisection on output label-correcting algorithm
- Efficient alternative: k-shortest paths (parallel)

Numerical Experiments

Numerical Experiments



Online Optimal Departure time

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Therefore, we also consider an online version of the problem:

$$t_u^* := \sup\{t \geqslant u \mid \mathbb{P}\big(t + T_t \leqslant M \mid B(u)\big) \geqslant \eta\}$$

Online vs Offline Optimal Departure time

For a range of on-time arrival probabilities η, we look at the difference in minutes for the online and offline departure time:

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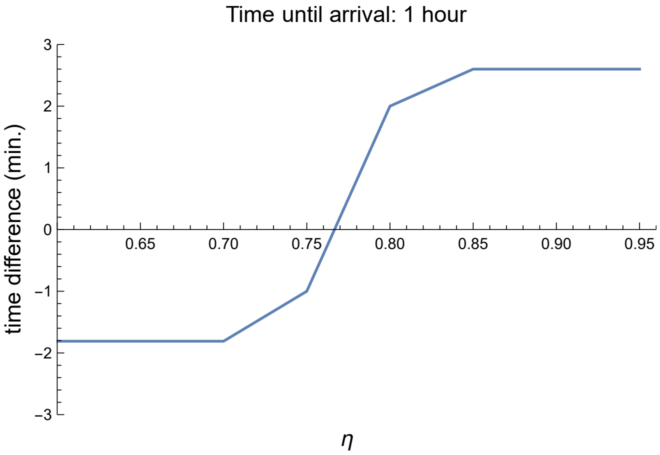

Outline

1. Markovian velocity model
2. Find the optimal route
3. **Determine the optimal departure time**
4. Related and other work

N. Levering, M. Boon, M. Mandjes, and R. Núñez-Queija
A framework for efficient dynamic routing under stochastically varying conditions
Transportation Research Part B: Methodological (2022), 160, 97-124.

R. Kamphuis, N. Levering, and M. Mandjes
Optimal departure-time advice in road networks with stochastic disruptions
Under review (Pre-print, arXiv:2208.14516 [math.OC]).

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MVM: Operationalization

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Estimating probability distributions of travel times by fitting a Markovian velocity model
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Idea:

- When using the MVM, it is assumed that Q and $v_a(B(t))$ are known
- How can these be found for a given highway network?
- Present methodology, with lists of incidents and a data base of recorded speeds as input
- Proof of concept in Dutch highway network

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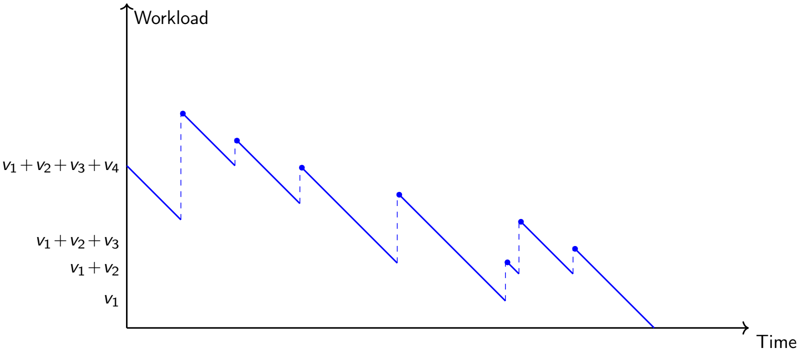
Road traffic streams

N. Levering and R. Núñez-Queija
Input rate control in stochastic road traffic networks: Effective bandwidths
Under review.

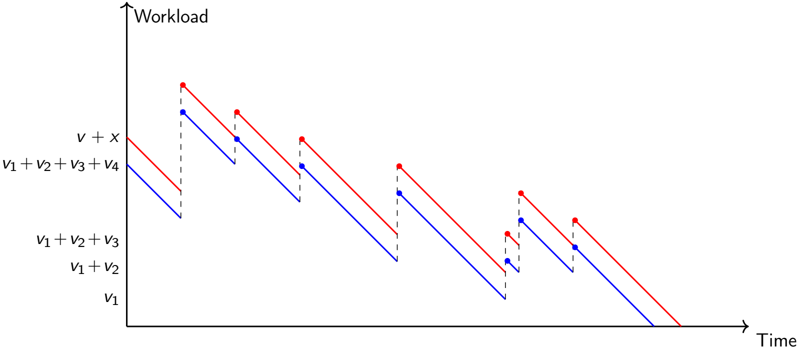
R. Kamphuis, N. Levering, and M. Mandjes
Optimal routing advice in highway networks with stochastic fundamental diagram dynamics
Working paper.

Queueing Theory

Queueing Theory

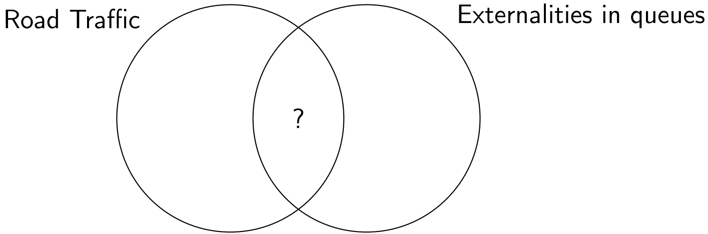


Queueing Theory



Queueing Theory

R. Jacobovic, N. Levering, and O. Boxma
Externalities in the M/G/1 queue: LCFS-PR versus FCFS
Queueing Systems (in press).


Road Traffic
Externalities in queues
?

Thank you for the attention!