Tailoring Policy Gradient to Product-Form Queueing Systems

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Work in Progress

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Reinforcement Learning

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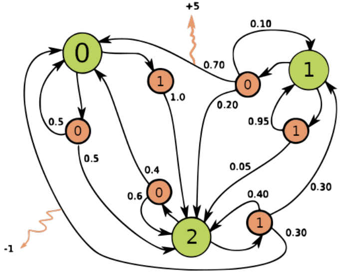

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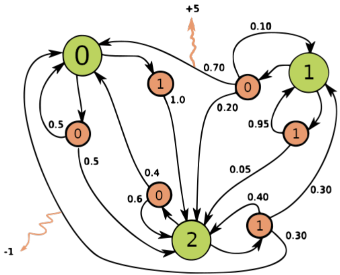

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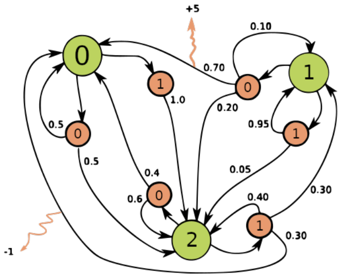

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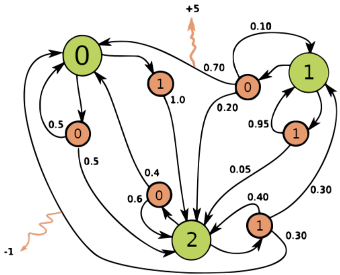

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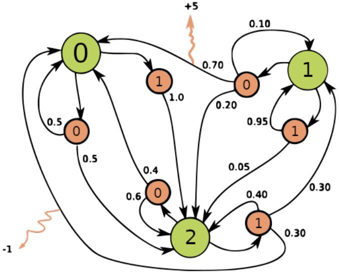

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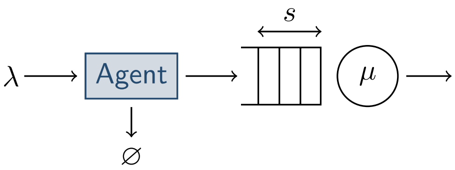
- $S \sim p(\cdot|\theta)$ stationary distribution of $(S_t, t = 0, 1, 2, \ldots)$
  $(S, A, R) \sim$ stationary distribution of $((S_t, A_t, R_{t+1}), t = 0, 1, 2, \ldots)$

Example 1: M/M/1 Queue with Admission Control

- Arrival rate $\lambda > 0$, service rate $\mu > \lambda$
- State: queue length $s \in \{0, 1, 2, \dots\}$
- Actions: accept or reject
- Reward α per accepted job
- Holding cost η per job per time unit



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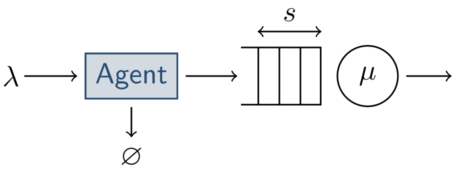

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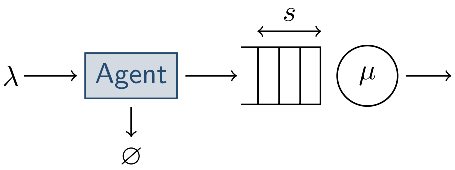

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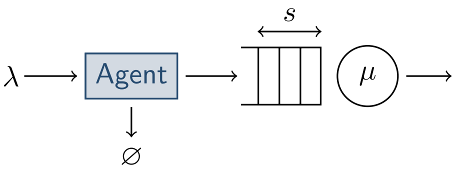

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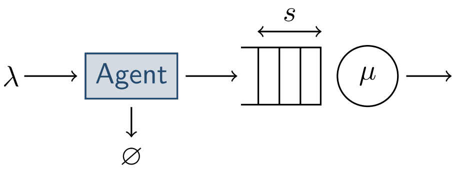

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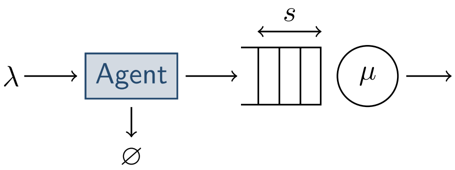

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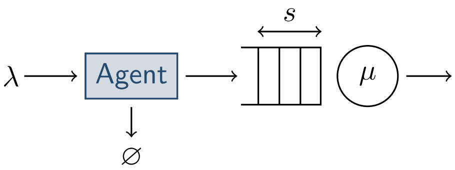

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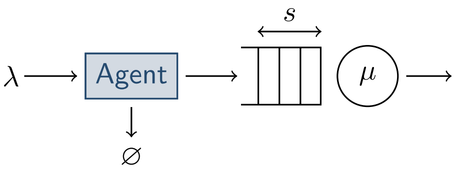

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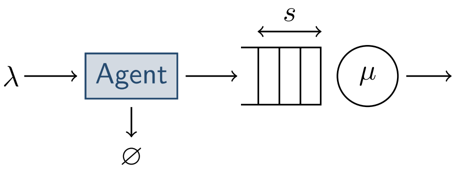

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- Jobs arrive as a Poisson process with rate λ
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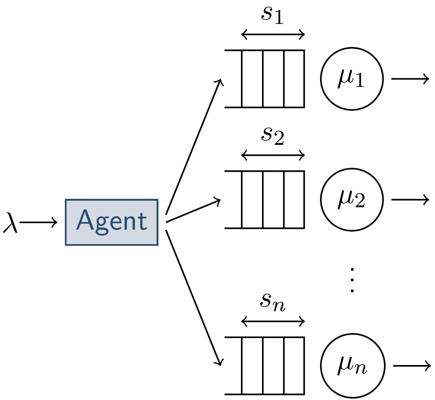

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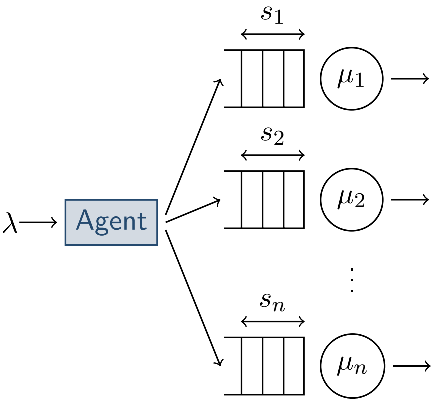

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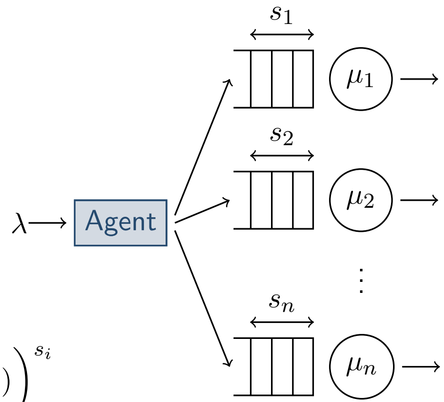

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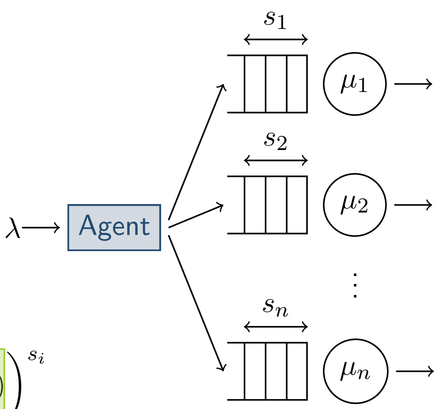

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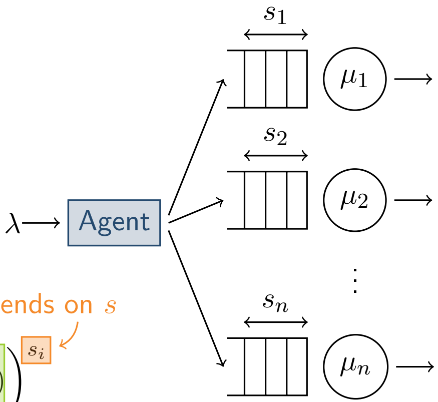

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- Main contributions:
  1. Product-form distributions as exponential families
  2. Score-aware gradient estimator (SAGE)
  3. SAGE-based policy-gradient algorithm
  4. Convergence result (work in progress)

① Product-Form Distributions as Exponential Families

- **Product-form** distribution

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- **Partition function** Z ⟶ **Log-partition function** $\log Z$

$$Z(\theta) = \sum_s \prod_{i=1}^{n} \rho_i(\theta)^{x_i(s)}$$

$$\log Z(\theta) = \log \left(\sum_s \mathrm{e}^{\langle \log \rho(\theta), x(s) \rangle} \right)$$

② Score-aware gradient estimator (SAGE)

- The **score** is the gradient of the log-likelihood with respect to the parameter vector:

$$\text{“Likelihood”} = p(s|\theta) \quad \rightarrow \quad \text{“Score”} = \nabla_\theta \log p(s|\theta) = (\partial_{\theta_i} \log p(s|\theta), i = 1, 2, \ldots, n)$$

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$$\text{“Likelihood”} = p(s|\theta) \quad \rightarrow \quad \text{“Score”} = \nabla_\theta \log p(s|\theta) = (\partial_{\theta_i} \log p(s|\theta), i = 1, 2, \dots, n)$$

Theorem

Recalling that $(S, A, R) \sim$ *stationary distribution of* $((S_t, A_t, R_{t+1}), t = 0, 1, 2, \dots)$*, we have*

$$\partial_{\theta_i} \log p(s|\theta) = \langle \partial_{\theta_i} \log \rho(\theta), x(s) - \mathbb{E}[x(S)] \rangle,$$
$$\partial_{\theta_i} J(\theta) = \langle \partial_{\theta_i} \log \rho(\theta), \mathrm{Cov}[x(S), R] \rangle + \mathbb{E}[\partial_{\theta_i} \log \pi(A|S, \theta) R].$$

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$$\begin{aligned} \partial_{\theta_i} \log p(s|\theta) &= \langle \partial_{\theta_i} \log \rho(\theta), x(s) - \mathbb{E}[x(S)] \rangle, \\ \partial_{\theta_i} J(\theta) &= \langle \partial_{\theta_i} \log \rho(\theta), \mathrm{Cov}[x(S), R] \rangle + \mathbb{E}[\partial_{\theta_i} \log \pi(A|S, \theta) R]. \end{aligned}$$

- **Main take-away:** This gives us an estimator for $\nabla_\theta J(\theta) = (\partial_{\theta_i} J(\theta), i = 1, \ldots, n)$.

③ SAGE-Based Policy-Gradient Algorithm

- Typical **policy-gradient algorithm**:

1: Initialize S_0 and θ_0
2: **for** $t = 0, 1, 2, \dots$ **do**
3: Sample $A_t \sim \pi(\cdot|S_t, \theta_t)$
4: Take action A_t and observe S_{t+1}, R_{t+1}
5: Estimate $[\![\nabla_\theta J(\theta_t)]\!]$ using the history $S_0, \theta_0, A_0, R_1, \dots, S_t, \theta_t, A_t, R_{t+1}, S_{t+1}$
6: Update $\theta_{t+1} \leftarrow \theta_t + \alpha [\![\nabla_\theta J(\theta_t)]\!]$
7: **end for**

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- **Actor-critic** applies the policy-gradient theorem (Sutton and Barto, 2018):

$$[\![\partial_{\theta_i} J(\theta_t)]\!] \leftarrow ([\![\mathbb{E}[R]]\!] - [\![v]\!](S_t))\partial_{\theta_i} \log \pi(A_t|S_t, \theta_t).$$

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- We instead estimate $[\![\nabla_\theta J(\theta_t)]\!]$ with a **score-aware gradient estimator (SAGE)**:

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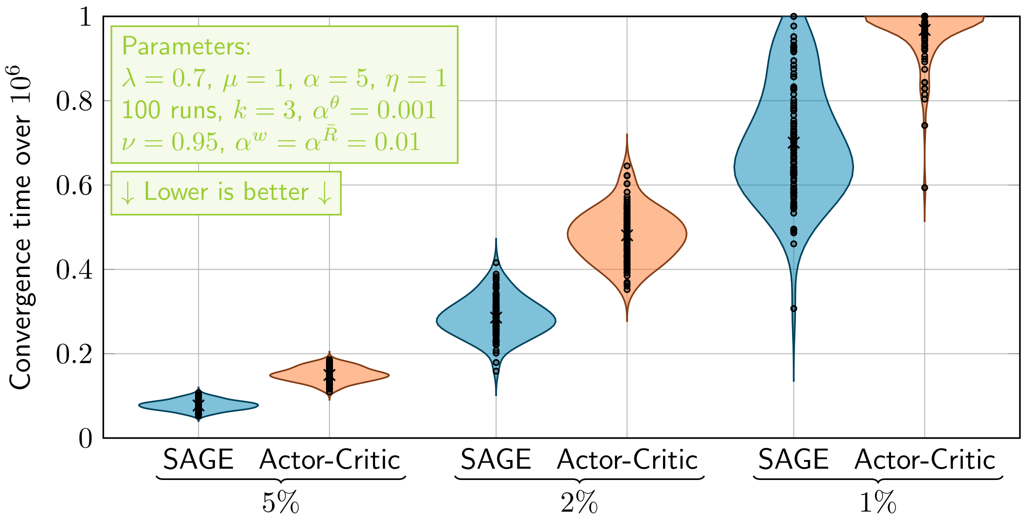
Example 1: M/M/1 Queue with Admission Control



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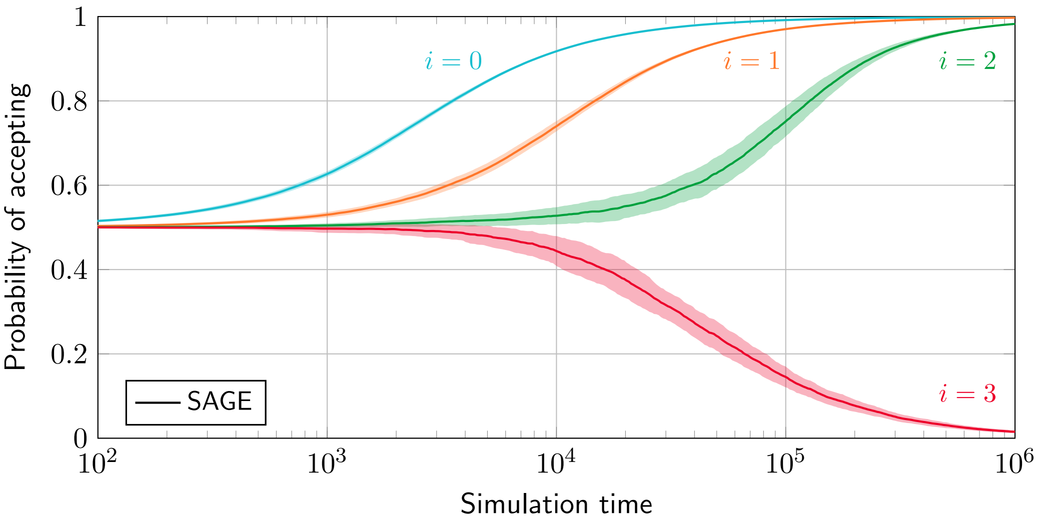

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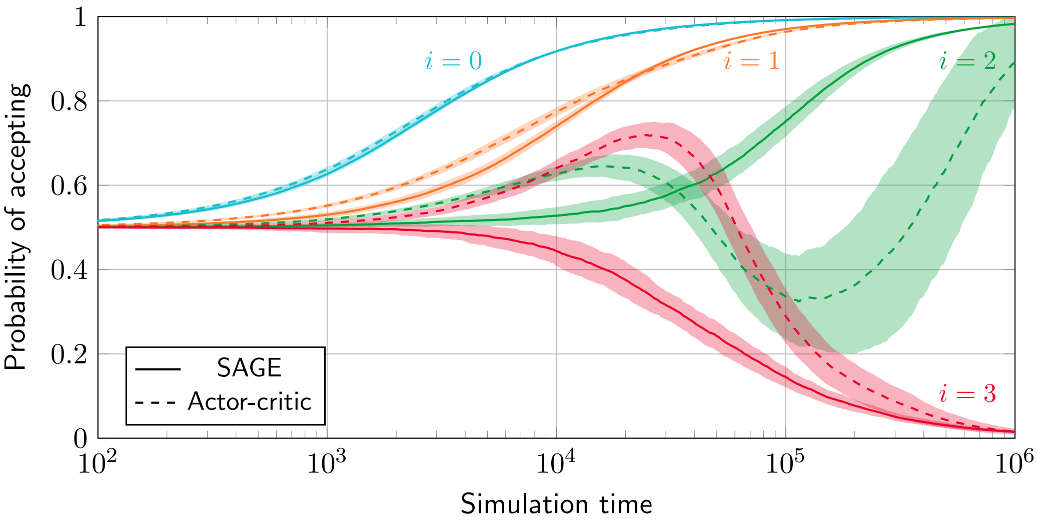
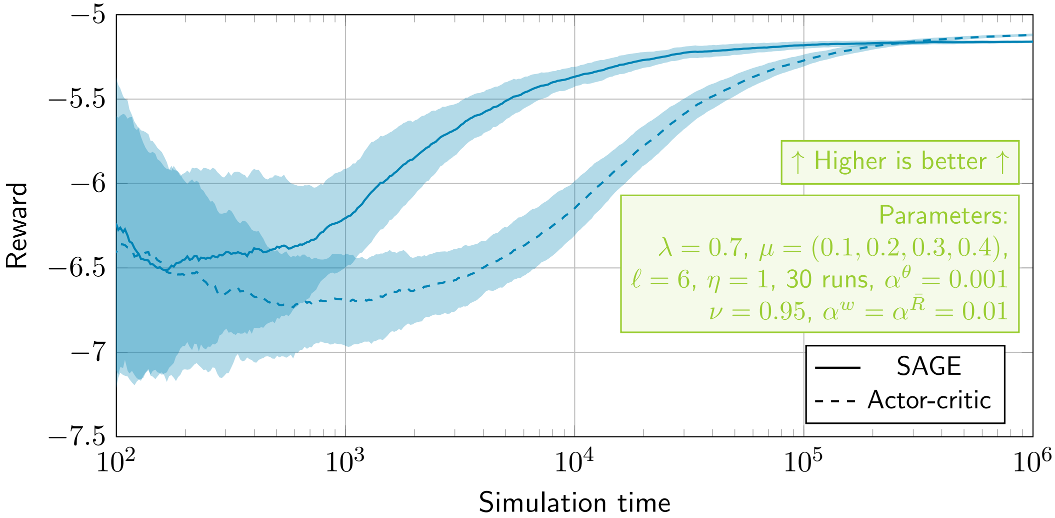


Example 2: Load Balancing



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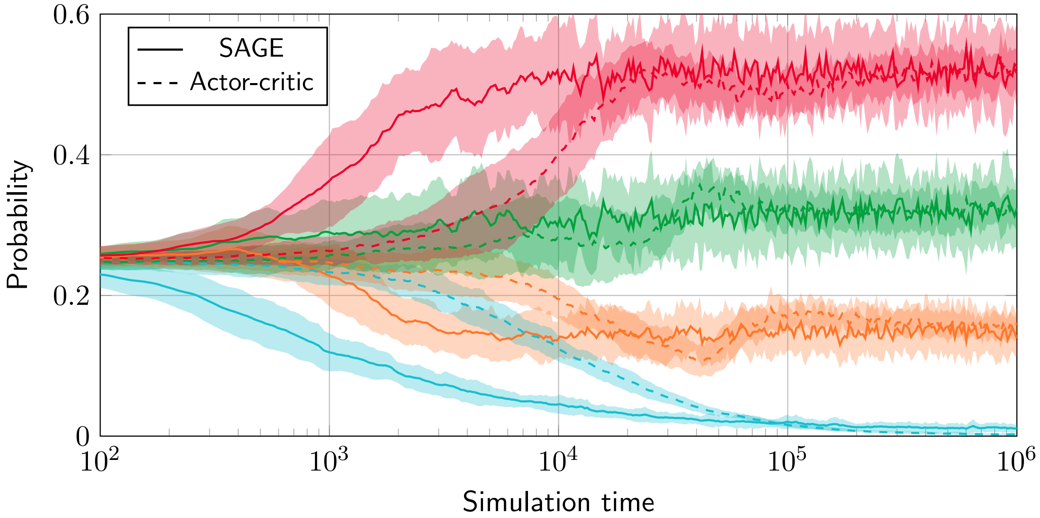

Conclusion

- **Main contributions**
  1. Product-form distributions as exponential families
  2. Score-aware gradient estimator (SAGE)
  3. SAGE-based policy-gradient algorithm
  4. Convergence result (work in progress)

Product-form stationary distribution

$$\log p(s|\theta) = \langle \log \rho(\theta), x(s) \rangle - \log Z(\theta)$$

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- **Future research directions**
  - Run extensive numerical results on more challenging examples.

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Score-aware gradient estimator (SAGE)

- **Future research directions**
  - Run extensive numerical results on more challenging examples.
  - Better estimators for covariance and expectation: robust covariance, etc.
  - Applications to (queueing) systems where the stationary distribution is known only *up to a multiplicative constant*.